

Remarks on a Chern-Simons-like coupling

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We consider the static quantum potential for a gauge theory which includes a light massive vector field interacting with the familiar $U(1)_{QED}$ photon via a Chern-Simons-like coupling, by using the gauge-invariant, but path-dependent, variables formalism. An exactly screening phase is then obtained, which displays a marked departure of a qualitative nature from massive axionic electrodynamics. The above static potential profile is similar to that encountered in axionic electrodynamics consisting of a massless axion-like field, as well as to that encountered in the coupling between the familiar $U(1)_{QED}$ photon and a second massive gauge field living in the so-called $U(1)_h$ hidden-sector, inside a superconducting box.

I. INTRODUCTION

The existence of axion-like particles and light extra hidden $U(1)$ gauge bosons have been proposed in many investigations of extensions of the Standard Model (SM), in order to explain cosmological and astrophysical results^{1–13}. Let us recall here that the axion-like scenario can be qualitatively understood by the existence of light pseudoscalar bosons ϕ ("axions"), with a coupling to two photons. In this case the interaction term in the effective Lagrangian has the form $\mathcal{L}_I = -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}\phi$, where $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$. However, the crucial feature of axionic electrodynamics is the mass generation due to the breaking of rotational invariance induced by a classical background configuration of the gauge field strength [14], which leads to confining potentials in the presence of nontrivial constant expectation values for the gauge field strength [15].

In this perspective, in recent times a different extension of the SM has been considered [16, 17], in order to overcome difficulties from the CAST [18] experiment, which represents an alternative to the axion-like scenario. It is the so-called Chern-Simons-like coupling scenario, which includes a light massive vector field interacting with the familiar $U(1)_{QED}$ photon via a Chern-Simons-like coupling. As a result, it was argued that this new model reproduces the effects of rotation of the polarization plane. In other terms, on the one hand both the Chern-Simons-like coupling and axionic electrodynamics models are quite different and, on the other hand, have optical-like features which they share. Therefore, is rather justifiable to have some additional understanding of the physical consequences presented by this new scenario (Chern-Simons-like coupling scenario), from a somewhat different perspective.

Our goal in this letter is precisely to examine the impact of a light massive vector field in the Chern-Simons-like coupling scenario through a proper study of the concepts of screening and confinement. In this perspective, we recall that the effective quantum potential between two static charges is a key tool to study the equivalence among models, which, otherwise are only suggested by other, very formal approaches. Accordingly, our analysis reveals both expected and unexpected features of the model under consideration. To accomplish our analysis, we use the gauge-invariant but path-dependent variables formalism along the lines of Ref. 19 – 22, which is a physically-based alternative to the usual Wilson loop approach and a preliminary version of this work has appeared before [23].

II. INTERACTION ENERGY

As mentioned above, the gauge theory we are considering describes the interaction between the familiar massive $U(1)_{QED}$ photon with a light massive vector field via a Chern-Simons-like coupling. In this case the corresponding theory is governed by the Lagrangian density [16, 17] :

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2(A) - \frac{1}{4}F_{\mu\nu}^2(B) + \frac{m_\gamma^2}{2}A_\mu^2 + \frac{m_B^2}{2}B_\mu^2 - \frac{\kappa}{2}\varepsilon^{\mu\nu\lambda\rho}A_\mu B_\nu F_{\lambda\rho}(A), \quad (1)$$

where m_γ is the mass of the photon, and m_B represents the mass for the gauge boson B . Notice that this alternative theory exhibits an effective mass for the component of the photon along the direction of the external magnetic field,

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exactly as it happens with axionic electrodynamics. Nevertheless, the Lagrangian above is not so a-priori as Eq. (1) seems to indicate. Actually, it comes out as the unitary-gauge-fixed version of a more fundamental and full gauge invariant action thoroughly discussed in the works [16] and [17].

Next, if we consider the model in the limit of a very heavy B -field ($m_B \gg m_\gamma$) and we are bound to energies much below m_B , we are allowed to integrate over B_μ and to speak about an effective model for the A_μ -field. This can be readily accomplished by means of the path integral formulation of the generating functional associated to Eq.(1), where the B_μ -field appears at most quadratically. By then shifting it according to the expression

$$B_\mu \equiv \tilde{B}_\mu + \frac{\kappa}{2} \frac{1}{(\Delta + m_B^2)} \left[\eta_{\mu\nu} + \frac{\partial_\mu \partial_\nu}{m_B^2} \right] \varepsilon^{\mu\lambda\rho\nu} A_\mu F_{\lambda\rho}(A), \quad (2)$$

and carrying out the Gaussian integration over the \tilde{B}_μ -field, we are lead to the effective Lagrangian for A_μ , as it follows below:

$$\begin{aligned} \mathcal{L}_{eff} = & -\frac{1}{4} F_{\mu\nu}^2 + \frac{m_\gamma^2}{2} A_\mu A^\mu + \frac{\kappa^2}{4} A_\alpha F_{\beta\gamma} \frac{1}{(\Delta + m_B^2)} A^\alpha F^{\beta\gamma} \\ & + \frac{\kappa^2}{2} A_\alpha F_{\beta\gamma} \frac{1}{(\Delta + m_B^2)} A^\gamma F^{\alpha\beta} + \frac{\kappa^2}{8} \tilde{F}^{\alpha\beta} F_{\alpha\beta} \frac{1}{m_B^2 (\Delta + m_B^2)} \tilde{F}^{\gamma\delta} F_{\gamma\delta}, \end{aligned} \quad (3)$$

where $\tilde{F}_{\mu\nu} \equiv 1/2 \varepsilon_{\mu\nu\lambda\rho} F^{\lambda\rho}$. One should notice that no gauge ambiguity is present in the process of integrating out B_μ , for Eq.(1) is already a gauge-fixed Lagrangian, as we have pointed out in connection to Ref. 16. Also, we observe that the system described by the Lagrangian (3) is a system with non-local time derivatives. However, we stress that this paper is aimed at studying the static potential of the theory (3), so that Δ can be replaced by $-\nabla^2$. For notational convenience we have maintained Δ , but it should be borne in mind that this paper essentially deals with the static case. Thus, the canonical quantization of this theory from the Hamiltonian point of view follows straightforwardly, as we will show it below.

Now, if we wish to study quantum properties of the electromagnetic field in the presence of external electric and magnetic fields, we should split the A_μ -field as the sum of a classical background, $\langle A_\mu \rangle$, and a small quantum fluctuation, a_μ , namely: $A_\mu = \langle A_\mu \rangle + a_\mu$. Therefore the previous Lagrangian density, up to quadratic terms in the fluctuations, is also expressed as

$$\begin{aligned} \mathcal{L}_{eff} = & -\frac{1}{4} f_{\mu\nu} \Omega f^{\mu\nu} + \frac{1}{2} a_\mu M^2 a^\mu - \frac{\kappa^2}{2} f_{\gamma\beta} \langle A^\gamma \rangle \frac{1}{(\Delta + m_B^2)} \langle A_\alpha \rangle f^{\alpha\beta} \\ & + \frac{\kappa^2}{8} f_{\mu\nu} v^{\mu\nu} \frac{1}{m_B^2 (\Delta + m_B^2)} v^{\lambda\rho} f_{\lambda\rho} - \frac{\kappa^2}{2} \left(\varepsilon^{jk0i} v_{0i} \langle A^l \rangle a_l \frac{1}{(\Delta + m_B^2)} f_{jk} \right) \\ & - \kappa^2 \left(\varepsilon^{jk0i} v_{0i} \langle A_j \rangle a^m \frac{1}{(\Delta + m_B^2)} f_{km} \right) + \kappa^2 \left(\varepsilon^{jk0i} v_{0i} \langle A^m \rangle a_k \frac{1}{(\Delta + m_B^2)} f_{jm} \right), \end{aligned} \quad (4)$$

where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, and $\Delta \equiv \partial_\mu \partial^\mu$. $\Omega \equiv 1 - \kappa^2 \frac{\langle A^i \rangle \langle A_i \rangle}{(\Delta + m_B^2)}$, and $M^2 \equiv m_\gamma^2 + \frac{\kappa^2}{2} \frac{v_{0i} v^{0i}}{(\Delta + m_B^2)}$. In the above Lagrangian we have considered the $v^{0i} \neq 0$ and $v^{ij} = 0$ case (referred to as the magnetic one in what follows), and simplified our notation by setting $\varepsilon^{\mu\nu\alpha\beta} \langle F_{\mu\nu} \rangle \equiv v^{\alpha\beta}$. As a result, the Lagrangian (4) becomes a Maxwell-Proca-like theory with a manifestly Lorentz violating term.

This effective theory provide us with a suitable starting point to study the interaction energy. However, before proceeding with the determination of the energy, it is necessary to restore the gauge invariance in (4). Standard techniques for constrained systems then lead to the following effective Lagrangian:

$$\begin{aligned} \mathcal{L}_{eff} = & -\frac{1}{4} f_{\mu\nu} \left[\frac{(\Delta^2 + a^2 \Delta + b^2)}{\Delta (\Delta + m_B^2)} \right] f^{\mu\nu} - \langle A^i \rangle f_{i0} \frac{1}{(\Delta + m_B^2)} \langle A_k \rangle f^{k0} \\ & - \frac{\kappa^2}{2} f_{ki} \langle A^k \rangle \frac{1}{(\Delta + m_B^2)} \langle A_l \rangle f^{li} + \frac{\kappa^2}{8} v^{0i} f_{0i} \frac{1}{m_B^2 (\Delta + m_B^2)} v^{0k} f_{0k}, \end{aligned} \quad (5)$$

where $a^2 \equiv m_B^2 + m_\gamma^2 \left(1 - \kappa^2 \frac{\langle A_k \rangle \langle A^k \rangle}{m_\gamma^2} \right)$, and $b^2 = m_\gamma^2 \left(m_B^2 - \frac{\kappa^2 v^2}{2} \right)$. To get the above theory we have ignored the last three terms in (4) because it add nothing to the static potential calculation, as we will show it below. Consequently,

the new effective action (5) provide us with a suitable starting point to study the interaction energy without loss of physical content.

Having established the new effective Lagrangian, we can now compute the interaction energy. To this end, we first consider the Hamiltonian framework of this new effective theory. The canonical momenta read $\Pi^\mu = -\left(\frac{\Delta^2 + a^2 \Delta + b^2}{\Delta(\Delta + m_B^2)}\right) f^{0\mu} + \kappa^2 \frac{\langle A^\mu \rangle \langle A_k \rangle}{(\Delta + m_B^2)} f^{k0} + \frac{\kappa^2}{4} \frac{v^{0\mu}}{m_B^2} \frac{1}{(\Delta + m_B^2)} v^{0k} f_{0k}$, and one immediately identifies the usual primary constraint $\Pi^i = -\left(\frac{\Delta^2 + a^2 \Delta + b^2}{\Delta(\Delta + m_B^2)}\right) f^{0i} + \kappa^2 \frac{\langle A^i \rangle \langle A_k \rangle}{(\Delta + m_B^2)} f^{k0} + \frac{\kappa^2}{4} \frac{v^{0i}}{m_B^2} \frac{1}{(\Delta + m_B^2)} v^{0k} f_{0k}$. Therefore the canonical Hamiltonian takes the form

$$\begin{aligned} H_C = & \int d^3x \left\{ -a_0 \partial_i \Pi^i + \frac{1}{2} B^i \frac{(\Delta^2 + a^2 \Delta + b^2)}{\Delta(\Delta + m_B^2)} B^i \right\} - \frac{1}{2} \int d^3x \Pi_i \frac{(\Delta + m_B^2)}{(\Delta + a^2 + \frac{b^2}{\Delta})} \Pi^i \\ & + \frac{\kappa^2}{2} \int d^3x \Pi_i \langle A^i \rangle \frac{(\Delta + m_B^2)}{(\Delta + a^2 + \frac{b^2}{\Delta})^2} \langle A^k \rangle \Pi_k + \frac{\kappa^2}{2} \int d^3x f_{ki} \langle A^k \rangle \frac{1}{(\Delta + m_B^2)} \langle A_l \rangle f^{li} \\ & + \frac{\kappa^2 \mathbf{v}^2}{8m_B^2} \int d^3x \Pi_i \frac{(\Delta + m_B^2)}{(\Delta + a^2 + \frac{b^2}{\Delta})^2} \Pi^i, \end{aligned} \quad (6)$$

where $a^2 = m_B^2 + m_\gamma^2 + \kappa^2 \langle \mathbf{A} \rangle^2$ and $b^2 = m_\gamma^2 m_B^2 + \frac{\kappa^2}{2} \mathbf{v}^2$. Since our energies are all much below m_B , it is consistent with our considerations to neglect $\kappa^2 \langle \mathbf{A} \rangle^2$ with respect to m_B^2 . This implies that a^2 and b^2 should be taken as: $a^2 = m_B^2$ and $b^2 = m_\gamma^2 m_B^2 + \frac{\kappa^2}{2} \mathbf{v}^2$.

Temporal conservation of the primary constraint Π_0 leads to the secondary constraint $\Gamma_1(x) \equiv \partial_i \Pi^i = 0$. It is also possible to verify that no further constraints are generated by this theory. The extended Hamiltonian that generates translations in time then reads $H = H_C + \int d^2x (c_0(x) \Pi_0(x) + c_1(x) \Gamma_1(x))$, where $c_0(x)$ and $c_1(x)$ are arbitrary Lagrange multipliers. Moreover, it follows from this Hamiltonian that $\dot{a}_0(x) = [a_0(x), H] = c_0(x)$, which is completely arbitrary. Since $\Pi^0 = 0$ always, neither a^0 and Π^0 are of interest in describing the system and may be discarded from the theory. If a new arbitrary coefficient $c(x) = c_1(x) - A_0(x)$ is introduced the Hamiltonian may be rewritten as

$$\begin{aligned} H = & \int d^3x \left\{ c(x) \partial_i \Pi^i + \frac{1}{2} B^i \frac{(\Delta^2 + a^2 \Delta + b^2)}{\Delta(\Delta + m_B^2)} B^i \right\} - \frac{1}{2} \int d^3x \Pi_i \frac{(\Delta + m_B^2)}{(\Delta + a^2 + \frac{b^2}{\Delta})} \Pi^i \\ & + \frac{\kappa^2}{2} \int d^3x \Pi_i \langle A^i \rangle \frac{(\Delta + m_B^2)}{(\Delta + a^2 + \frac{b^2}{\Delta})^2} \langle A^k \rangle \Pi_k + \frac{\kappa^2}{2} \int d^3x f_{ki} \langle A^k \rangle \frac{1}{(\Delta + m_B^2)} \langle A_l \rangle f^{li} \\ & + \frac{\kappa^2 \mathbf{v}^2}{8m_B^2} \int d^3x \Pi_i \frac{(\Delta + m_B^2)}{(\Delta + a^2 + \frac{b^2}{\Delta})^2} \Pi^i. \end{aligned} \quad (7)$$

Since there is one first class constraint $\Gamma_1(x)$ (Gauss' law), we choose one gauge fixing condition that will make the full set of constraints becomes second class. We choose the gauge fixing condition to correspond to

$$\Gamma_2(x) \equiv \int_{C_{\xi x}} dz^\nu a_\nu(z) \equiv \int_0^1 d\lambda x^i a_i(\lambda x) = 0, \quad (8)$$

where λ ($0 \leq \lambda \leq 1$) is the parameter describing the spacelike straight path $x^i = \xi^i + \lambda(x - \xi)^i$, and ξ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\xi^i = 0$. The choice (8) leads to the Poincaré gauge [25, 26]. As a consequence, we can now write down the only nonvanishing Dirac bracket for the canonical variables

$$\{a_i(x), \Pi^j(y)\}^* = \delta_i^j \delta^{(3)}(x - y) - \partial_i^x \int_0^1 d\lambda x^j \delta^{(3)}(\lambda x - y). \quad (9)$$

We pass now to the calculation of the interaction energy, where a fermion is localized at the origin $\mathbf{0}$ and an antifermion at \mathbf{y} . For this purpose, we will calculate the expectation value of the energy operator H in the physical

state $|\Phi\rangle$. In this context, we recall that the physical state $|\Phi\rangle$ can be written as

$$|\Phi\rangle \equiv |\bar{\Psi}(\mathbf{y}) \Psi(\mathbf{0})\rangle = \bar{\psi}(\mathbf{y}) \exp\left(iq \int_{\mathbf{0}}^{\mathbf{y}} dz^i a_i(z)\right) \psi(\mathbf{0}) |0\rangle, \quad (10)$$

where $|0\rangle$ is the physical vacuum state. The line integral is along a spacelike path starting at $\mathbf{0}$ and ending at \mathbf{y} , on a fixed time slice.

Returning now to our problem on hand, we compute the expectation value of H in the physical state $|\Phi\rangle$. Taking into account the above Hamiltonian structure, we observe that

$$\Pi_i(x) |\bar{\Psi}(\mathbf{y}) \Psi(\mathbf{y}')\rangle = \bar{\Psi}(\mathbf{y}) \Psi(\mathbf{y}') \Pi_i(x) |0\rangle + q \int_{\mathbf{y}}^{\mathbf{y}'} dz_i \delta^{(3)}(\mathbf{z} - \mathbf{x}) |\Phi\rangle. \quad (11)$$

Having made this observation and since the fermions are taken to be infinitely massive (static sources), we can substitute Δ by $-\nabla^2$ in Eq. (7). Therefore, the expectation value $\langle H \rangle_{\Phi}$ is expressed as

$$\langle H \rangle_{\Phi} = \langle H \rangle_0 + \langle H \rangle_{\Phi}^{(1)} + \langle H \rangle_{\Phi}^{(2)}, \quad (12)$$

where $\langle H \rangle_0 = \langle 0 | H | 0 \rangle$. The $\langle H \rangle_{\Phi}^{(1)}$ and $\langle H \rangle_{\Phi}^{(2)}$ terms are given by

$$\langle H \rangle_{\Phi}^{(1)} = -\frac{b^2 B}{2} \int d^3x \langle \Phi | \Pi_i \frac{\nabla^2}{(\nabla^2 - M_1^2)} \Pi^i | \Phi \rangle + \frac{b^2 B}{2} \int d^3x \langle \Phi | \Pi_i \frac{M_2^2}{M_1^2} \frac{\nabla^2}{(\nabla^2 - M_2^2)} \Pi^i | \Phi \rangle, \quad (13)$$

and

$$\langle H \rangle_{\Phi}^{(2)} = m_B^2 b^2 B \int d^3x \langle \Phi | \Pi_i \frac{1}{(\nabla^2 - M_1^2)} \Pi^i | \Phi \rangle - m_B^2 b^2 B \int d^3x \langle \Phi | \Pi_i \frac{M_2^2}{M_1^2} \frac{1}{(\nabla^2 - M_2^2)} \Pi^i | \Phi \rangle, \quad (14)$$

where $B = \frac{1}{M_2^2(M_2^2 - M_1^2)}$, $M_1^2 \equiv \frac{a^2}{2} \left[1 + \sqrt{1 - \frac{4b^2}{a^4}}\right]$, and $M_2^2 \equiv \frac{a^2}{2} \left[1 - \sqrt{1 - \frac{4b^2}{a^4}}\right]$.

We have neglected the terms in (7) where $\left(\Delta + a^2 + \frac{b^2}{\Delta}\right)^2$ appears in the denominator, the reason being that we wish to compute an interparticle potential, which expresses the effects of photons exchange in the low-energy (or low-frequency) limit. Therefore, these terms we are mentioning are suppressed in view of the presence of higher power of the frequency in the denominator. Another important point to be highlighted in our discussion comes from the expressions for M_1^2 and M_2^2 . Our treatment is only valid under the assumption that $a^4 > 4b^2$. However, this condition is equivalent to taking $\kappa^2 \mathbf{v}^2 < \frac{m_B^4}{2}$, which is perfectly compatible with our approximation. So, we restrict ourselves to an external magnetic field such that $|\mathbf{v}| < \frac{m_B^2}{2\kappa^2}$.

Following our earlier procedure^{19–24}, we see that the potential for two opposite charges located at $\mathbf{0}$ and \mathbf{y} takes the form

$$\begin{aligned} V = & -\frac{q^2}{4\pi} \left[\frac{1}{2} \left(1 - \frac{3}{\sqrt{1 - 4(m_\gamma^2/m_B^2 + 2\kappa^2 B^2/m_B^4)}} \right) \frac{1}{L} \exp \left[\left(-\frac{m_B}{\sqrt{2}} \sqrt{1 + \sqrt{1 - 4(m_\gamma^2/m_B^2 + 2\kappa^2 B^2/m_B^4)}} \right) L \right] \right. \\ & \left. + \frac{1}{2} \left(1 + \frac{3}{\sqrt{1 - 4(m_\gamma^2/m_B^2 + 2\kappa^2 B^2/m_B^4)}} \right) \frac{1}{L} \exp \left[\left(-\frac{m_B}{\sqrt{2}} \sqrt{1 - \sqrt{1 - 4(m_\gamma^2/m_B^2 + 2\kappa^2 B^2/m_B^4)}} \right) L \right] \right]. \quad (15) \end{aligned}$$

Here \mathcal{B} represents the external magnetic field and $|\mathbf{y}| \equiv L$. This result immediately shows that the theory under consideration describes an exactly screening phase. It is worthwhile stressing that the choice of the gauge is in this development really arbitrary. This then implies that we would obtain exactly the same result in any gauge. It is appropriate to observe here that by considering the limit m_γ and $\kappa \rightarrow 0$, we obtain a theory of two independent uncoupled $U(1)$ gauge bosons, one of which is massless. In such a case, one can easily verify that the static potential is a Yukawa-like correction to the usual static Coulomb potential.

III. FINAL REMARKS

In summary, we have considered the confinement versus screening issue for a gauge theory which includes a light massive vector field interacting with the familiar $U(1)_{QED}$ photon via a Chern-Simons-like coupling. Interestingly enough, expression (15) displays a marked departure from the result of axionic electrodynamics. As already expressed, axionic electrodynamics has a different structure which is reflected in a confining piece, which is not present in the Chern-Simons-like coupling scenario. Nevertheless, the above static potential profile is similar to that encountered in axionic electrodynamics consisting of a massless axion-like field. In fact, the linear confining potential seems to be associated only with massive axion-like particles. As opposed to non-Abelian axionic electrodynamics that, even in the massless case, displays a confining potential. In addition, we also mention that the above static potential profile is analogous to that encountered in the coupling between the familiar massless electromagnetism $U(1)_{QED}$ and a hidden-sector $U(1)_h$ inside a superconducting box [24]. In this connection it becomes of interest, to recall that an experiment for searching for extra hidden-sector $U(1)$ gauge bosons with small gauge kinetic mixing (χ) with the ordinary photon (in the laboratory) has been proposed in [13]. Basically, this experiment consists in putting a sensitive magnetometer inside a superconducting shielding, which in turn is placed inside a strong magnetic field. In this case, it was argued that photon - hidden photon oscillations would allow to penetrate the superconductor and a magnetic field would register on the magnetometer, in contrast with the usual electrodynamics where the magnetic field cannot penetrate the superconductor. In this setup the magnetometer measures the field strength which is proportional to χ^2 . In this sense the present work could be of interest for searching for bounds of the gauge kinetic mixing (χ) along the lines of expression (15). Thus, we have established a new connection between extensions of the Standard Model (SM) such as axion-like particles and light extra $U(1)$ gauge bosons. Accordingly, the benefit of considering the present approach is to provide unifications among different models, as well as exploiting the equivalence in explicit calculations, as we have illustrated in the course of this work.

Finally, it should be noted that by substituting B_μ by $\partial_\mu\phi$ in (1), the theory under consideration assumes the form [27]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{m_\gamma^2}{2}A_\mu^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\kappa}{2m_B}\varepsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\phi, \quad (16)$$

which is similar to axionic electrodynamics. In fact, it is worth recalling here that axionic electrodynamics is described by [15]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g}{8}\phi\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m_A^2}{2}\phi^2, \quad (17)$$

hence we see that both theories are quite different.

Next, after performing the integration over ϕ in (16), the effective Lagrangian density reads

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{m_\gamma^2}{2}A_\mu^2 - \frac{\kappa^2}{8m_B^2}\varepsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\frac{1}{\nabla^2}\varepsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}. \quad (18)$$

This expression can now be rewritten as

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}^2 + \frac{m_\gamma^2}{2}a_\mu^2 - \frac{\kappa^2}{2m_B^2}\varepsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}\rangle\varepsilon^{\lambda\rho\gamma\delta}\langle F_{\lambda\rho}\rangle f_{\alpha\beta}\frac{1}{\nabla^2}f_{\gamma\delta}, \quad (19)$$

where $\langle F_{\mu\nu}\rangle$ represents the constant classical background. Here $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ describes fluctuations around the background. The above Lagrangian arose after using $\varepsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}\rangle\langle F_{\alpha\beta}\rangle = 0$ (which holds for a pure electric or a pure magnetic background). By introducing the notation $\varepsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}\rangle \equiv v^{\alpha\beta}$ and $\varepsilon^{\rho\sigma\gamma\delta}\langle F_{\rho\sigma}\rangle \equiv v^{\gamma\delta}$, expression (19) then becomes

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}^2 + \frac{m_\gamma^2}{2}a_\mu^2 - \frac{\kappa^2}{2m_B^2}v^{\alpha\beta}f_{\alpha\beta}\frac{1}{\nabla^2}v^{\gamma\delta}f_{\gamma\delta}, \quad (20)$$

where the tensor $v^{\alpha\beta}$ is not arbitrary, but satisfies $\varepsilon^{\mu\nu\alpha\beta}v_{\mu\nu}v_{\alpha\beta} = 0$. With this in view, we now proceed to calculate the interaction energy in the $v^{0i} \neq 0$ and $v^{ij} = 0$ case (referred to as the magnetic one in what follows).

Following the same steps employed for obtaining (15), the static potential is expressed as

$$V = -\frac{q^2}{4\pi L}\exp\left[-\left(\sqrt{m_\gamma^2 + 4\kappa^2\mathcal{B}^2/m_B^2}\right)L\right], \quad (21)$$

where \mathcal{B} represents the external magnetic field. We observe that in the limit $m_\gamma \rightarrow 0$, axionic electrodynamics experiences mass generation induced by an external magnetic field. In this way the theory describes a screening phase, as we have just seen above. Evidently, by considering the limit $\kappa \rightarrow 0$, we obtain a Proca-like theory.

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